

## Boolean Algebra – Operations and Constants

- $A \text{ AND } B = A \wedge B = AB$
- $A \text{ OR } B = A \vee B = A+B$
- $\text{NOT } A = \neg A = A'$
- $\text{TRUE} = T = 1$
- $\text{FALSE} = F = 0$

## Boolean Algebra - Identities

- $A \cdot \text{True} = A$
- $A \cdot \text{False} = \text{False}$
- $A \cdot A = A$
- $(A')' = A$
- $A + A' = \text{True}$
- $A \cdot A' = \text{False}$
- $A + \text{True} = \text{True}$
- $A + \text{False} = A$
- $A + A = A$

## Commutative, Associative, and Distributive Laws

- $AB = BA$  (Commutative)
- $A + B = B + A$
- $A(BC) = (AB)C$  (Associative)
- $A + (B + C) = (A + B) + C$
- $A(B + C) = (AB) + (AC)$  (Distributive)
- $A + (BC) = (A + B)(A + C)$

## DeMorgan's Laws

- $(A + B)' = A'B'$
- $(AB)' = A' + B'$

## Example: Proving Identities

- Using truth tables, prove:
- $A + A' = \text{True}$                        $A \cdot A' = \text{False}$

A	A'	A + A'
F		
T		

A	A'	A • A'
F		
T		

## (One of the) Associative Laws

- Using truth tables, prove  
 $A (B C) = (A B) C$

A	B	C	BC	A(BC)	AB	(AB)C
F	F	F				
F	F	T				
F	T	F				
F	T	T				
T	F	F				
T	F	T				
T	T	F				
T	T	T				

## (One of the) Distributive Laws

- Using truth tables, prove

$$A(B + C) = (AB) + (AC)$$

A	B	C	B+C	A(B+C)	AB	AC	(AB)+(AC)
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

## Proving DeMorgan's Laws (a)

- Using truth tables, prove  $(A + B)' = A'B'$

A	B	A+B	(A+B)'
F	F		
F	T		
T	F		
T	T		

A	B	A'	B'	A'B'
F	F			
F	T			
T	F			
T	T			

## Proving DeMorgan's Laws (b)

- Prove the 2<sup>nd</sup> of DeMorgan's Laws:

$$(AB)' = A' + B'$$

A	B	AB	(AB)'
F	F		
F	T		
T	F		
T	T		

A	B	A'	B'	A' + B'
F	F			
F	T			
T	F			
T	T			

## Exercise: $A(A + B)$

A	B	A + B	A(A + B)

What have we proved in this table?

## Exercise: Boolean Algebra

- Exercise - Using the Distributive Property, Identities, and your result from the previous exercise, prove:
  - $A + (AB) = A$
  - $A + (AB)$   
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## Exercise: Using DeMorgan's Laws

- Exercise – Using Boolean Algebra, including DeMorgan's Laws, prove:
  - $(A'B)' = A + B'$
  - $(A'B)'$   
=  $((A')B)'$             (add parentheses)  
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