CS 215 Algorithms
Sample Test 1

Name:

You must show your work for full credit. Guessing or simply “knowing” the answer will only be worth partial credit. You may make assumptions to remove the floor and ceiling functions. While there are 105 points on the exam, the most credit you can get is 100 points for 100 per cent. Write your name and student number on each sheet you turn in. You are allowed one sheet of notes (front and back). You do not need to turn in your notes sheet.

1) (5 pts) What is the sum from $i = 0$ to $n$ of $2^i$ for $n = 5$?

2) (5 pts) If you are given: $\log_c a$ and $\log_c b$, how would you compute $\log_b a$?

3) (5 pts) Simplify $\sum_{i=0}^{k} c \cdot 2^i$ if $c$ is a constant.

4) Short answer (10 pts each):
   a) Comment on the statement, “I have an algorithm so complicated that no computer could solve it.”
b) List two uses for a loop invariant
   i)  
   ii) 

c) create a “c” assert statement corresponding to the following invariant (hint: Think about how to step through an array):

\[ A[i] < A[i + 1], i = 1, 2, \ldots, j \]

5) Show the following are correct or incorrect:
   a) (5 pts) \[42n^2 - 6n = \Theta(n^3)\]
b) (5pts) $10n^2 + 9 = \Theta(n)$

6) Given the following formula, $T(n) = c_1n + 3T\left(\frac{n}{3}\right)$, what can you tell me about the related procedure (5pts)? Draw the recursion tree (5pts).
7) The following program purports to find the maximum element in an array.

Algorithm 1 Max(A[], n)
1: procedure Max(A[], n)
2:    i=2; j=1;
3:    while (i <= n) do
4:       if (A[i] >= A[j]) then
5:          j=i;
6:       end if
7:    i=i+1;
8: end while
9: end procedure

a) Develop a postcondition for Max in terms of A, n, j (10 pts)

b) Formulate a loop invariant for the while loop (hint: Think about how Max explores the loop) (10 pts)

c) Show that your loop invariant is correct with respect to the program post conditions (10 pts)
8) (10pts) The SPAMCO computer corporation claims their new computer will run 100 times faster than that of their competitor, Dogbert, Inc. Given two algorithms A and B with running times as follows:
\[ t_A(n) = 5000n \]
\[ t_B(n) = 10n^2 \]

For what values of \( n \), if any, will the Dogbert computer run algorithm A faster than the SPAMCO computer will run algorithm B?

9) Prove that the solution of following recurrence for \( n \) a power of 2:

\[
T(n) = \begin{cases} 
2T(n/2) + cn & n > 1 \\
1 & \text{otherwise}
\end{cases}
\]

is \( T(n) = O(n \log_2 n) \)