Boolean Algebra

Logical Statements

- A proposition that may or may not be true:
  - **Today is Monday.**
  - **Today is Sunday.**
  - **It is raining.**

Compound Statements

- More complicated expressions can be built from simpler ones:
  - **Today is Monday AND it is raining.**
  - **Today is Sunday OR it is NOT raining**
  - **Today is Monday OR today is NOT Monday**
    - (This is a tautology)
  - **Today is Monday AND today is NOT Monday**
    - (This is a contradiction)
- The expression *as a whole* is either true or false.

Boolean Algebra

- Boolean Algebra allows us to formalize this sort of reasoning.
- Boolean variables may take one of only two possible values: TRUE or FALSE.
  - **(or, equivalently, 1 or 0)**
- Arithmetic operators: + - * /
- Logical operators - AND, OR, NOT, XOR

Logical Operators

- A AND B - True only when both A and B are true.
- A OR B - True unless both A and B are false.
- NOT A - True when A is false. False when A is true.
- A XOR B - True when either A or B are true, but not when both are true.

A	B	A AND B
F	F	F
F	T	F
T	F	F
T	T	T

A	B	A OR B
F	F	F
F	T	T
T	F	T
T	T	T

A	NOT A
F	T
T	F

Writing AND, OR, NOT

- A AND B = A ^ B = AB
- A OR B = A v B = A+B
- NOT A = ¬A = A’
- TRUE = T = 1
- FALSE = F = 0

Boolean Algebra

- The = in Boolean Algebra indicates equivalence
- Two statements are equivalent if they have exactly the same conditions for being true. (More in a second)
- For example,
  - **True = True**
  - **A = A**
  - **(AB)' = (A' + B')**

Truth Tables

- Provide an exhaustive approach to describing when some statement is true (or false)

Truth Table

M	R	M'	R'	MR	M + R
F	F				
F	T				
T	F				
T	T				

Truth Table

M	R	M’	R’	MR	M + R
F	F	T	T		
F	T	T	F		
T	F	F	T		
T	T	F	F		

Truth Table

M	R	M’	R’	MR	M + R
F	F	T	T	F	
F	T	T	F	F	
T	F	F	T	F	
T	T	F	F	T	

Truth Table

M	R	M’	R’	MR	M + R
F	F	T	T	F	F
F	T	T	F	F	T
T	F	F	T	F	T
T	T	F	F	T	T

Exercise

- Write the truth table for (A + B) B

Exercise: (A + B) B

A	B	A + B	(A + B) B

Solution to (A + B) B

A	B	A + B	(A + B) B
F	F	F	F
F	T	T	T
T	F	T	F
T	T	T	T

Note: Truth Tables can be used to ***prove*** equivalencies. What have we proved in this table?

Solution to (A + B) B

A	B	A + B	(A + B) B
F	F	F	F
F	T	T	T
T	F	T	F
T	T	T	T

Note: Truth Tables can be used to ***prove*** equivalencies. What have we proved in this table?

(A + B) B = B

Boolean Algebra - Identities

- A AND ? = A

  A AND True =?= A

  A AND False =?= A

Boolean Algebra - Identities

- A AND ? = A

  A AND True = A

A	True	A AND True
F	T	F
T	T	T

- So,what about
  A AND False ?

Boolean Algebra - Identities

- A AND ? = A

  A AND True = A

A	True	A AND True
F	T	F
T	T	T

- So,what about

  A AND False ?

  A AND False = False

A	False	A AND False
F	F	F
T	F	F

Boolean Algebra - Identities

- A OR ? = A

  A OR True =?= A

  A OR False =?= A

Boolean Algebra - Identities

- A OR ? = A

  A OR False = A

A	False	A OR False
F	F	F
T	F	T

- So,what about
  A OR True?

Boolean Algebra - Identities

- A OR ? = A

  A OR False = A

A	False	A OR False
F	F	F
T	F	T

- So,what about
  A OR True?
  A OR True = True

A	True	A OR True
F	T	T
T	T	T

Boolean Algebra - Identities

- $A \bullet True = A$
- $A \bullet False = False$
- $A \bullet A = A$
- $A + True = True$
- $A + False = A$
- $A + A = A$

- $(A')' = A$
- $A + A' = True$
- $A \bullet A' = False$

Commutative, Associative, and Distributive Laws

- $AB = BA$ (Commutative)
- $A + B = B + A$
- $A(BC) = (AB)C$ (Associative)
- $A + (B + C) = (A + B) + C$
- $A (B + C) = (AB) + (AC)$ (Distributive)
- $A + (BC) = (A + B) (A + C)$

DeMorgan's Laws

- $(A + B)' = A'B'$
- $(AB)' = A' + B'$